APPLICATION OF THE ORTHOTROPIC PLATE THEORY TO GARAGE DECK DIMENSIONING

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SUMMARY

This paper focuses on the application of orthotropic plate bending theory to stiffened plating. Schade’s design charts for rectangular plates are extended to the case where the boundary contour is clamped, which is almost totally incomplete in the afore mentioned charts.

A numerical solution for the clamped orthotropic plate equation is obtained. The Rayleigh-Ritz method is adopted, expressing the vertical displacement field by a double cosine trigonometric series, whose coefficients are determined by solving a linear equation system. Numerical results are proposed as design charts similar to those ones by Schade. In particular, each chart is relative to one of the non-dimensional coefficients identifying the plate response; each curve of any chart is relative to a given value of the torsional parameter $\eta_t$, in a range comprised between 0 and 1, and is function of the virtual aspect ratio $\rho$, comprised between 1 and 8, so that the asymptotic behaviour of the orthotropic plate for $\rho \rightarrow \infty$ is clearly shown.

Finally, some numerical applications relative to ro-ro decks are presented, in order to evaluate the accuracy and the capability of the proposed technique for stiffened deck analysis. Obtained results are examined in order to draw a usable procedure for dimensioning deck primary supporting members, taking into account the interaction of the two orthogonal beam sets.

1. INTRODUCTION

Schade, 1942, proposed some practical general design curves, based on the “orthotropic plate” theory, in order to obtain a rapid, but accurate, dimensioning of plating stiffeners. Schade considered four types of boundary conditions for the orthotropic partial differential equation: all edges rigidly supported but not fixed; both short edges clamped, both long edges supported; both long edges clamped, both short edges supported; all edges clamped. The last case with all edges clamped was left almost totally incomplete. The few data useful for this boundary condition were taken from Timoshenko et al., 1959, and Young, 1940, as given for the isotropic plate only for the torsional coefficient value $\eta_t = 1$ and for a range of the virtual aspect ratio $\rho$ comprised between 1 and 2.

In this work a numerical solution of the clamped orthotropic plate equation is obtained. Numerical results are presented in a series of charts similar to those ones given by Schade.

Obtained results are applied to the analysis of ro-ro garage decks, taking into due consideration the characteristic distribution of wheeled loads. In particular, two typical structural configurations have been examined and results are discussed aiming at obtaining a simple procedure for primary supporting member dimensioning.

2. A NUMERICAL SOLUTION OF THE CLAMPED RECTANGULAR ORTHOTROPIC PLATE EQUATION

Orthotropic plate theory refers to materials which have different elastic properties along two orthogonal directions. In order to apply this theory to panels having a finite number of stiffeners, it is necessary to idealize the structure, assuming that the structural properties of the stiffeners may be approximated by their average values, which are assumed to be distributed uniformly over the width and the length of the plate.

Referring to the coordinate system of fig.1, the deflection field in bending is governed by the so called Huber’s differential equation:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p(x, y) \quad (1)$$

where:
- $D_x$ is the unit flexural rigidity around the $y$ axis;
• \( D_y \) is the unit flexural rigidity around the \( x \) axis;
• \( H = \eta \sqrt{D_x D_y} \) according to the definition by Schade;
• \( p \) is the pressure load over the surface.

It is noticed that the behavior of the isotropic plate with the same flexural rigidities in all directions is a special case of the orthotropic plate problem. Indicating with \( n \) the normal external to the plate contour, a numerical solution of the orthotropic plate equation with the boundary conditions:

\[
w = 0 \quad \text{and} \quad \frac{\partial w}{\partial n} = 0
\]

along all edges is presented. Now, as the plate domain is rectangular, the boundary conditions (2) become:

\[
w = 0 \quad \text{and} \quad \frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = 0
\]

So any displacement function, satisfying the boundary conditions (3), must belong, with the first order derivatives, to the function space with compact support in \( \Omega \), i.e. \( w \in C_0^1(\Omega) \), having denoted by \( \Omega \) the function domain.

Now, two solution methods are available: the double cosine series and the Hencky’s method. The second one is well known to converge quickly but does pose some difficulties with regard to programming due to over/underflow problems in the evaluation of hyperbolic trigonometric functions with large arguments. The double cosine series method, instead, is devoid of the over/underflow issue but is known to converge very slowly.

If \( a \) and \( b \) are the plate lengths in the \( x \) and \( y \) directions respectively, the vertical displacement field may be expressed by means of the following double cosine series:

\[
w(x, y) = \sum_{m=1}^{M} \sum_{n=1}^{N} \left( 1 - \cos \frac{2\pi m x}{a} \right) \left( 1 - \cos \frac{2\pi n y}{b} \right) c_{m,n} \quad (4)
\]

whose terms satisfy the boundary conditions (2). The unknown coefficients \( c_{m,n} \) may be determined using the Rayleigh-Ritz method, searching for the minimum of a variational functional. Now, denoting by \( u \) and \( f \) two classes of functions belonging to a Hilbert Space, for linear differential operators as:

\[
\ell u = f \quad (5)
\]

that are auto-added and defined positive, it is possible to find a numerical solution of the equation (5) searching for the stationary point of the functional:

\[
F(u) = \frac{1}{2} \int_\Omega \ell u \cdot ud\Omega - \int_\Omega f \cdot ud\Omega
\]

The linear operator \( \ell \) of the equation (5) is auto-added if, \( \forall u(x, y) \in L^2(\Omega) \) and \( \forall v(x, y) \in L^2(\Omega) \) satisfying the boundary conditions (3), it is verified that:

\[
\int_\Omega \ell u \cdot vd\Omega = \int_\Omega \ell v \cdot ud\Omega \quad (7)
\]

where \( \Omega \) is an open set of \( \mathbb{R}^2 \).

Now, let us consider the generalized integration by parts formula:

\[
\int_\Omega (uD_y)v dt = \int_\Omega (uv \circ n) d\sigma - \int_\Omega (vD_y u) dt \quad (8)
\]

where \( n \) is the versor of the normal external to \( \partial A \) and \( \varepsilon_i \) is the versor of \( i \) axis. First of all, in order to apply the equation (8), it is necessary to suppose that \( \Omega \subset \mathbb{R}^2 \) is a regular domain, i.e. that it is a limited domain with one or more contours that have to be generally regular curves. In the case under examination, as \( \Omega \) is a rectangular domain, these conditions are certainly verified. Furthermore, as \( w \in C_0^1(\Omega) \), it derives that:

\[
\int_\Omega (uD_y)v dt = -\int_\Omega (vD_y u) dt \quad (9)
\]

but, thanks to the boundary conditions (3), it is also possible to verify that:

\[
\int_\Omega (uD^\alpha v) dt = (-1)^{\| \alpha \|} \int_\Omega (vD^\alpha u) dt \quad (10)
\]

whatever is the multi-index \( \alpha = (\alpha_1, \alpha_2) \) with \( |\alpha| \leq 4 \), having denoted by \( |\alpha| = \alpha_1 + \alpha_2 \) the sum of the derivation number respect to the first variable and the second one, respectively. From equation (10) it is immediately verified the condition (7), as the partial differential operators are of even order.

Furthermore the linear operator \( \ell \) is defined positive if it is verified that:

\[
\int_\Omega \ell u \cdot ud\Omega > 0 \quad (11)
\]

Applying the generalized integration by parts formula, the integral (11) becomes:

\[
\int_\Omega \left[ D_x \left( \frac{\partial^2 w}{\partial x^2} \right) + 2H \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 + D_y \left( \frac{\partial^2 w}{\partial y^2} \right) \right] dA > 0 \quad \forall w \neq 0 \quad (12)
\]

If \( w = 0 \), thanks to the continuity of the displacement function, it would result:

\[
\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial y^2} = 0 \quad \forall (x, y) \in \Omega \quad (13)
\]
so obtaining:
\[
\begin{align*}
\frac{\partial w}{\partial x} &= \text{const.} \\
\frac{\partial w}{\partial y} &= \text{const.}
\end{align*}
\]  
(14)

and then, thanks to the continuity on the boundary:
\[
\frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = 0 \quad \forall (x, y) \in \partial \Omega
\]
(15)

From eq. (15) it would result:
\[
u = \text{const.} \quad \forall (x, y) \in \partial \Omega
\]
(16)

and then, thanks to the continuity on the boundary:
\[
u = 0 \quad \forall (x, y) \in \partial \Omega
\]
(17)

So the condition (11) must be necessarily verified.
In order to find the coefficients of eq. (3), it is imposed that the functional (5) is stationary:
\[
\frac{\partial F}{\partial w_{mn}} = 0
\]
(18)

In this case the functional (6) is written as follows:
\[
\Pi(w) = \frac{1}{2} \int \sum_{n} \left( D_{x} \frac{\partial^{2} w}{\partial x^{2}} + 2Hw \frac{\partial^{2} w}{\partial y x} + D_{y} \frac{\partial^{2} w}{\partial y^{2}} \right) dA +
\]
\[
- \int \Pi dw
\]
(19)

Applying the generalized integration by parts formula the functional (19) becomes:
\[
\Pi(w) = \frac{1}{2} \int \sum_{n} \left( D_{x} \frac{\partial^{2} w}{\partial x^{2}} + 2Hw \frac{\partial^{2} w}{\partial y x} + D_{y} \frac{\partial^{2} w}{\partial y^{2}} \right) dA +
\]
\[
- \int \Pi dw
\]
(20)

To carry out the computations, it is convenient to use the following coordinate transformations:
\[
x = a\xi \quad ; \quad 0 \leq \xi \leq 1
\]
(21.1)
\[
y = b\eta \quad ; \quad 0 \leq \eta \leq 1
\]
(21.2)

so that the series is given in nondimensional coordinates:
\[
w(\xi, \eta) = \sum_{m,n} \left( 1 - \cos 2\pi m \xi \right) \left( 1 - \cos 2\pi n \eta \right) w_{mn}
\]
(22)

Then the functional is written in the form:
\[
\tilde{\Pi}(w) = \Pi(w)_{ab} =
\]
\[
\frac{1}{2} \int \sum_{n} \left[ D_{x} \left( \frac{\partial^{2} w}{\partial x^{2}} \right)^{2} + 2H \frac{\partial^{2} w}{\partial y x} \frac{\partial^{2} w}{\partial y^{2}} + D_{y} \left( \frac{\partial^{2} w}{\partial y^{2}} \right)^{2} \right] d\xi d\eta +
\]
\[
- \int \Pi dw
\]
(23)

and the stationary point is obtained imposing the \(MxN\) equations system:
\[
\frac{\partial}{\partial w_{mn}} \tilde{\Pi}(w) = 0 \quad \text{for} \ m=1\ldots M \ ; \ n=1\ldots N
\]
(24)

So, considering \(p\) as uniformly distributed, the generic equation, for \(m=\bar{m}\) and \(n=\bar{n}\), assumes the form:
\[
\frac{\partial}{\partial w_{m\bar{n}}} \tilde{\Pi}(w) = \int \int \left( 1 - \cos 2\pi m \xi \right) \left( 1 - \cos 2\pi n \eta \right) d\xi d\eta = 1
\]
(25)

As regards the second member of equation (25), it is certainly possible to write the partial differential operator under the integral sign, so obtaining:
\[
\int \int \frac{\partial}{\partial w_{m\bar{n}}} \Pi dw d\xi d\eta = \int \int \left( 1 - \cos 2\pi m \xi \right) \left( 1 - \cos 2\pi n \eta \right) d\xi d\eta = 1
\]
(26)

The first integral at the left hand side of the equation (25) becomes:
\[
\int \int \frac{\partial}{\partial w_{m\bar{n}}} \left( \frac{\partial^{2} w}{\partial \xi^{2}} \right) d\xi d\eta = \int \int \frac{\partial^{2} w}{\partial \xi^{2}} d\xi d\eta = 32\pi^{2} m \int \int \left( 1 - \cos 2\pi m \xi \right) \left( 1 - \cos 2\pi n \eta \right) d\xi d\eta = 0
\]
(27)

In a similar way, the third term becomes:
\[
\int \int \frac{\partial}{\partial w_{m\bar{n}}} \left( \frac{\partial^{2} w}{\partial \eta^{2}} \right) d\xi d\eta = 8\pi^{2} m \int \left( w_{m\bar{n}} + 2\sum_{n=1}^{N} w_{m,n} \right)
\]
(28)

Manipulating similarly the second term, it is obtained:
\[
\int \int \frac{\partial}{\partial w_{m\bar{n}}} \left( \frac{\partial^{2} w}{\partial \xi \partial \eta} \right) d\xi d\eta = 16\pi^{2} m \int \left( 1 - \cos 2\pi m \xi \right) \left( 1 - \cos 2\pi n \eta \right) d\xi d\eta +
\]
(29)
\[+16\pi^2 n^2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} b m^2 w_{m,n} \left[ (1 - \cos 2\pi m\xi) \cos 2\pi n\eta \xi \right] \cos 2\pi n\eta \eta \]
\[\cdot (1 - \cos 2\pi n\eta) \eta = 8\pi^2 m^2 n w_{m,n} \tag{29}\]

Introducing the expressions (27), (28), (29), the left hand side of equation (25) can be so expressed:
\[
\begin{align*}
&\left\{ D_x \left[ m^2 w_{m,n} + \sum_{m=1}^{\infty} 2m w_{m,n} \right] + D_y \left[ a^2 \right] \left[ n^2 w_{m,n} + \sum_{n=1}^{\infty} 2n w_{m,n} \right] + \\
&+ 2 \left( \frac{a}{b} \right)^2 H m n w_{m,n} \right\} 8\pi^4
\end{align*}
\]
\[\tag{30}\]

Introducing the torsional coefficient \(\eta\) and the virtual side ratio defined as:
\[\rho = \frac{a}{b} \sqrt{\frac{D_x}{D_y}} \tag{31}\]

the equation (25) can be so written:
\[
\begin{align*}
&4\pi^4 \left[ \frac{1}{\rho^2} \left( m^2 w_{m,n} + \sum_{m=1}^{\infty} 2m w_{m,n} \right) + n^2 w_{m,n} + \sum_{n=1}^{\infty} 2n w_{m,n} + \\
&+ 2\eta \frac{w_{m,n}}{\rho^2} m n w_{m,n} \right] = \frac{p b^3}{D_y}
\end{align*}
\[\tag{32}\]

Defining the non dimensional vertical displacements:
\[\delta = \frac{w}{p b^3} ; \quad \delta_{m,n} = \frac{w_{m,n}}{p b^3} \frac{D_y}{D_x} \tag{33}\]

the system finally becomes:
\[
\begin{align*}
&4\pi^4 \left[ \frac{1}{\rho^2} \left( m^2 \delta_{m,n} + \sum_{m=1}^{\infty} 2m \delta_{m,n} \right) + n^2 \delta_{m,n} + \sum_{n=1}^{\infty} 2n \delta_{m,n} + \\
&+ 2\eta \frac{\delta_{m,n}}{\rho^2} m n \delta_{m,n} \right] = 1 : m = 1...M \quad \text{and} \quad n = 1...N \tag{34}\]

Even if the double cosine trigonometric series converges very slowly, adopting sufficiently high values for \(M\) and \(N\), it is possible to obtain a very accurate solution of the equation (1) with the boundary conditions (2).

3. CHARACTERIZATION OF THE BEHAVIOUR OF CLAMPED STIFFENED PLATES

The orthotropic plate bending theory can be applied to the plate of fig. 1, reinforced by two systems of parallel beams spaced equal distances apart in the \(x\) and \(y\) directions. The rigidities \(D_x\) and \(D_y\) of equation (1) can be specialized as follows:
\[D_x = \frac{EI_x}{s_x} = E_i x \tag{35.1}\]
\[D_y = \frac{EI_y}{s_y} = E_i y \tag{35.2}\]

where \(E\) is the Young’s modulus and \(s_x\) (\(s_y\)) is the distance between girders (transverses). It is noticed that \(I_{xx}\) (\(I_{yy}\)) is the moment of inertia, including effective width \(b_{xx}\) (\(b_{yy}\)) of plating and the attached ordinary stiffeners of long (short) repeating primary supporting members, respect to the axis whose eccentricity from the reference plane \((z = 0)\) is to be determined as follows:
\[
\begin{align*}
&b_{xx} = \frac{1}{1 - \nu^2} \int (z - c_x) dz + \int (z - c_x) dy + \left( \frac{b_{xx}}{s_{xx}} - 1 \right) \int (z - c_x) dy = 0 \\
&b_{yy} = \frac{1}{1 - \nu^2} \int (z - c_y) dz + \int (z - c_x) dy + \left( \frac{b_{yy}}{s_{yy}} - 1 \right) \int (z - c_y) dy = 0
\end{align*}
\]
\[\tag{35.1}\tag{35.2}\]

where \(s_{xx}\) and \(s_{yy}\) are the spacings between ordinary stiffeners and \(P_x, A_i\) and \(a_i\) are the plating, the supporting member and the ordinary stiffener section areas, respectively. The moments of inertia have to be determined applying the following equations:
\[
\begin{align*}
&I_{xx} = \frac{b_{xx}}{1 - \nu^2} \int (z - c_x)^2 dz + \int (z - c_x)^2 dy + \left( \frac{b_{xx}}{s_{xx}} - 1 \right) \int (z - c_x)^2 dy = 0 \\
&I_{yy} = \frac{b_{yy}}{1 - \nu^2} \int (z - c_y)^2 dz + \int (z - c_y)^2 dy + \left( \frac{b_{yy}}{s_{yy}} - 1 \right) \int (z - c_y)^2 dy = 0
\end{align*}
\]
\[\tag{37.1}\tag{37.2}\]

The torsional coefficient \(\eta\) and the virtual side ratio \(\rho\) can be specialized according to Schade’s works:
\[
\begin{align*}
&\eta = \frac{i_{xx} i_{xy}}{i_{xy} i_{yy}} \tag{38.1} \\
&\rho = \frac{a}{b} \sqrt{\frac{i_{xy}}{i_{xx}}} \tag{38.2}\]
\]

where \(i_{xx}\) (\(i_{yy}\)) is the moment of inertia of effective breadth of plating working with long (short) supporting stiffeners per unit length. In the following \(r_{xy}\) (\(r_{yy}\)) is the vertical distance of the associated plating working with long (short) supporting stiffeners from the section neutral axis, while \(r_{xy}\) (\(r_{yy}\)) is the distance of the free flange from the section neutral axis.

The meaning of the two parameters is quite clear. In particular, the torsional coefficient \(\eta\), which lies between 0 and 1, exists because only the plating is subject to horizontal shear, while both the plating and stiffeners are subject to bending stress. Obviously \(\eta = 1\), and \(i_{xx} = i_{yy} = i_x = i_y\) represents the isotropic plate case. The virtual side ratio \(\rho\) is the plate side ratio modified in accordance with the unit stiffnesses in the two directions; as usual, it has been admitted that \(\rho\) is always equal to or greater than unity.
In the next the quantities represented in the diagrams are presented.

Deflection at center, fig. 2: the vertical displacement at the plate center \((\tau)=\xi=0.5\) is the maximum and is so expressed:

\[
w_{\text{max}} = k_w \frac{pb^4}{E_t y}
\]

(39.1)

where:

\[
k_w(\rho, \eta) = \sum_{m=1}^{N} \delta_{m,0}(1 - \cos \eta)(1 - \cos \eta)
\]

(39.2)

Edge bending stress in plating, fig. 3: these curves give the bending stress in the plating at the centers of edges where fixity exists. The stress at the center of such an edge may be treated as the maximum along that edge. The maximum stresses in the plating in the long and short directions respectively are:

\[
\sigma_{\text{plating}} = E \frac{1}{1 - \nu^2} \delta \frac{\partial^2 \delta}{\partial \xi^2} \frac{r_m}{E_t y} \frac{pb^4}{E_t y}
\]

(40.1)

\[
\sigma_{\text{plating}} = E \frac{1}{1 - \nu^2} \frac{b^2}{\partial \eta^2} \frac{\partial^2 \delta}{\partial \xi^2} \frac{r_m}{E_t y} \frac{pb^4}{E_t y}
\]

(40.2)

as along the edges it results:

\[
\left. \frac{\partial^2 \delta}{\partial \eta^2} \right|_{\eta=0} = 0 \quad \text{and} \quad \left. \frac{\partial^2 \delta}{\partial \xi^2} \right|_{\xi=0} = 0
\]

(41)

The equations (40.1) and (40.2) become:

\[
\sigma_{\text{plating}} = k_{\text{plating}}(\rho, \eta) \frac{pb^4 r_m}{\sqrt{1 - \nu^2} E_t y}
\]

(42.1)

\[
\sigma_{\text{plating}} = k_{\text{plating}}(\rho, \eta) \frac{pb^4 r_m}{i_y}
\]

(42.2)

where:

\[
k_{\text{plating}}(\rho, \eta) = \frac{4\pi^2}{1 - \nu^2} \sum_{m=1}^{N} \delta_{m,0}m^2(1 - \cos \eta)
\]

(43.1)

\[
k_{\text{plating}}(\rho, \eta) = \frac{4\pi^2}{1 - \nu^2} \sum_{m=1}^{N} \delta_{m,0}n^2(1 - \cos \eta)
\]

(43.2)

Edge bending stress in free flanges, fig. 4: these curves give the bending stress in the free flanges at the centers of edges where fixity exists. The stress at the center of such an edge may be treated as the maximum along that edge. The maximum stresses in the free flanges for girders and transverses are respectively:

\[
\sigma_{\text{SUP}} = -E \frac{1}{a^2} \frac{\partial^2 \delta}{\partial \xi^2} \frac{r_m}{E_t y} \frac{pb^4}{E_t y}
\]

(44.1)

\[
\sigma_{\text{SUP}} = -E \frac{1}{b^2} \frac{\partial^2 \delta}{\partial \eta^2} \frac{r_m}{E_t y} \frac{pb^4}{E_t y}
\]

(44.2)

The equations (44.1) and (44.2) can be re-written as follows:

\[
\sigma_{\text{SUP}} = -k_{\text{SUP}}(\rho, \eta) \frac{pb^4 r_m}{\sqrt{1 - \nu^2} E_t y}
\]

(45.1)

\[
\sigma_{\text{SUP}} = -k_{\text{SUP}}(\rho, \eta) \frac{pb^4 r_m}{i_y}
\]

(45.2)

where:

\[
k_{\text{SUP}}(\rho, \eta) = \frac{4\pi^2}{\rho^2} \sum_{m=1}^{N} \delta_{m,0}m^2(1 - \cos \eta)
\]

(46.1)

\[
k_{\text{SUP}}(\rho, \eta) = \frac{4\pi^2}{\rho^2} \sum_{m=1}^{N} \delta_{m,0}n^2(1 - \cos \eta)
\]

(46.2)

It is important to note that when \(\rho \to \infty\) \(k_{\text{SUP}}\) is substantially independent on \(\eta\) and is equal to \(\frac{1}{12}\) that is the beam theory value. Furthermore the curves show that for low values of \(\eta\) the maximum deflections and stresses parallel to the short direction occur at values of \(\rho\) between 1.5 and 2.0: this indicates that the long beams add to the load taken by the short beams, instead of helping to support it.

Bending stress in free flanges at center, fig. 5: these curves give the bending stress in the free flanges at the center of the panel in long and short directions respectively. The stresses:

\[
\sigma_{\text{CEN}} = -E \frac{1}{a^2} \frac{\partial^2 \delta}{\partial \xi^2} \frac{r_m}{E_t y} \frac{pb^4}{E_t y}
\]

(47.1)

\[
\sigma_{\text{CEN}} = -E \frac{1}{b^2} \frac{\partial^2 \delta}{\partial \eta^2} \frac{r_m}{E_t y} \frac{pb^4}{E_t y}
\]

(47.2)

can be so expressed:

\[
\sigma_{\text{CEN}} = k_{\text{CEN}}(\rho, \eta) \frac{pb^4 r_m}{\sqrt{1 - \nu^2} E_t y}
\]

(48.1)

\[
\sigma_{\text{CEN}} = k_{\text{CEN}}(\rho, \eta) \frac{pb^4 r_m}{i_y}
\]

(48.2)

where:

\[
k_{\text{CEN}}(\rho, \eta) = -\frac{4\pi^2}{\rho^2} \sum_{m=1}^{N} \delta_{m,0}m^2 \cos \eta \cos \eta(1 - \cos \eta)
\]

(49.1)
\[ k_\text{YCEM}(\rho, \eta) = -4\pi^2 \sum_{m=1}^{M} \sum_{n=1}^{N} \delta_{mn} \cos \pi m(1 - \cos \pi n) \] (49.2)

It is important to note that when \( \rho \to \infty \), \( k_{\text{YCEM}} \) is substantially independent on \( \eta \) and is equal to \( \frac{1}{24} \), that is the beam theory value.

In order to verify the goodness of the method, the following tables show a comparison between the values obtained applying the Rayleigh-Ritz method and the ones taken from Timoshenko et al., 1959, for the isotropic plate (\( \eta = 1.00 \)).

### Deflection at center

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>Timoshenko</th>
<th>( k_w \ (\eta = 1.00) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.00126</td>
<td>0.00126</td>
</tr>
<tr>
<td>1.20</td>
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<td>0.00172</td>
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<td>1.40</td>
<td>0.00207</td>
<td>0.00207</td>
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<td>1.60</td>
<td>0.00230</td>
<td>0.00230</td>
</tr>
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<td>1.80</td>
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<td>0.00245</td>
</tr>
<tr>
<td>2.00</td>
<td>0.00254</td>
<td>0.00253</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.00260</td>
<td>0.00260</td>
</tr>
</tbody>
</table>

**tab. 1**

### Edge bending moment in short direction

\[ (1-\nu^2)K_{\text{WSTP}} (\eta = 1.00) \]

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>Timoshenko</th>
<th>( k_w \ (\eta = 1.00) )</th>
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**tab. 2**

### Edge bending moment in long direction

\[ (1-\nu^2)K_{\text{WSTP}} (\eta = 1.00) \]

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<th>( k_w \ (\eta = 1.00) )</th>
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**tab. 3**

**fig. 2 - Deflection at center**
fig. 3 - Edge bending stress in plating

fig. 4 - Edge bending stress in free flanges
4. CONVERGENCE OF THE METHOD

In the following, the influence of the number of harmonics on $k$ values is shown. Particularly, assuming $\rho=5$ and $\eta=0.50$, $M=N$ has been varied from 5 up to 100, in order to obtain a number of harmonics comprised between 25 and 10000. If the number of harmonics is $> 4900$, i.e. $M=N > 70$, a good convergence in the assessment of $k$ values, and then of the proposed curves, is obtained for practical purposes, as it can be appreciated from fig. 6, 7, 8.
5. THE CASE OF DISCONTINUOUS LOADS

The partial differential equation (1) has been written with reference to a distributed normal pressure load which is a continuous function in the plate $\mathbb{R}$.

Let’s now suppose that $p \in L^1(\Omega)$, so that the set of discontinuity points has zero measure according to Lebesgue.

Let’s define with $\mathbb{R}_0 \subseteq \mathbb{R}$ the point set where $p$ is continuous and with $\mathbb{R}_1 \subseteq \mathbb{R} : m(\mathbb{R}_1) = 0$ the point set where $p$ is discontinuous.

The two subsets $\mathbb{R}_0$ and $\mathbb{R}_1$ define a partition of $\mathbb{R}$:

\[
\begin{aligned}
\mathbb{R}_0 \cup \mathbb{R}_1 &= \mathbb{R} \\
\mathbb{R}_0 \cap \mathbb{R}_1 &= \emptyset
\end{aligned}
\] (50)

Rigorously, as (1) is valid point by point only where $p$ is continuous, the functional (19) has to be extended only to the $\mathbb{R}_0$ domain. But, as $p$ is continuous almost everywhere in $\mathbb{R}$, the functional $w(\mathbb{R})$ can be extended to the entire $\mathbb{R}$ domain. It is noticed that, as $w \in L^1(\Omega)$, according to the Schwartz-Holder inequality, $P(w) = \int_{\mathbb{R}} w(\mathbb{R}) d\mathbb{R}$.

Moreover, as an integral extended to a set of zero measure is equal to zero according to Lebesgue, the following equalities hold:

\[
\Pi(w)|_{\mathbb{R}_0} = \Pi(w)|_{\mathbb{R}_0 \cup \mathbb{R}_1} = \Pi(w)|_{\mathbb{R}_1}
\] (51)

Then, it is possible to apply the equation (1) not only when the load function is continuous in $\mathbb{R}$, but also when it is continuous almost everywhere in $\mathbb{R}$, in both cases extending the functional (19) to the entire domain according to the identity (51).

The extension to load functions continuous almost everywhere according to Lebesgue is particularly useful when it is necessary to schematize the wheeled loads. In this case, in fact, the effective load distribution can be modelled as an equivalent pressure, transversally constant but longitudinally discontinuous:

\[ p_{eq}(\xi, \eta) = p, \quad \forall \xi \in [\alpha, \beta], \quad \forall \eta \in [0,1] \] (52)

6. THE EQUIVALENT PRESSURE FOR WHEELED LOADS

For primary supporting members subjected to wheeled loads, yielding checks have to be carried out considering a maximum pressure load, equivalent to the maximum vertical, static and dynamic, applied forces; the static part can be evaluated with the following relation, suggested by R.I.N.A., 2005:

\[ p_{eq.stat} = \frac{n_v Q_a}{ls} \left( 3 - \frac{X_1 + X_2}{s} \right) g \] (53)

in which it is assumed:

- $n_v$ = maximum number of vehicles located on the primary supporting member;
- $Q_a$ = maximum axle load in t;
- $X_1$ = minimum distance, in m, between two consecutive axles;
- $X_2$ = minimum distance, in m, between the axles of two consecutive vehicles;
- $s$ = span, in m, of the primary supporting members;
- $l$ = spacing, in m, of primary supporting members.

The maximum total equivalent pressure is the sum of the static term and the dynamic one and can be expressed in kN/m$^2$ as follows:

\[ p_{eq.max} = (1 + a_x) p_{eq.stat}. \] (54)

where $a_x$ is the ship vertical acceleration.

The following figure shows the origin of the formula (53).

The three wheels give the following contributions to eq. (53):

- \[ p_{eq.stat.1} = \frac{n_v Q_a}{ls} \left( s - \frac{X_1}{s} \right) g \]
- \[ p_{eq.stat.2} = \frac{n_v Q_a}{ls} \left( s - \frac{X_2}{s} \right) g \]

The equation (53) is valid only if an axle is located directly on a supporting member, but if this condition is not verified the previous relation can’t be directly applied. So, it is convenient to generalize the eq. (53) as follows:

\[ p_{eq.stat} = \frac{n_v}{ls} \sum_{i=1}^{n_v} Q_i \left( 1 - \frac{X_i}{s} \right) g \] (55)
where \( n_{ia} \) is the number of axles between \(-s\) and \( s\) and \( X_i\) is the distance of the \( i\)-th axle load from the considered supporting member. From eq. (55), the actual equivalent pressure \( p_i\), including inertial force, is obtained similarly to eq. (54).

In such a way it is possible to model the load distribution on the deck on the basis of axle loads and geometric characteristics of vehicles. As in this case the deck isn’t loaded by a uniform pressure load, but by a load function discontinuous at intervals, the integral at the second term of (25) has to be replaced as follows:

\[
2a^4 \frac{\partial}{\partial w_{max}} \int_{0}^{2\pi} p w d\xi d\eta =
\]

\[
= 2p_{eq.max}a^4 \sum_{i=1}^{n_F} \kappa_i \left( \int_{0}^{2\pi m} \left| \frac{1 - \cos 2\pi \eta}{2\pi} \right| d\eta \right) =
\]

\[
= 2p_{eq.max}a^4 \sum_{i=1}^{n_F} \kappa_i \left( \beta_i - \alpha_i \frac{\sin 2\pi \beta_i - \sin 2\pi \alpha_i}{2\pi} \right) (56)
\]

where \( n_F \) is the number of intervals where \( p \) is continuous, coinciding with the number of transverses, \( p_{eq.max} \) is the maximum equivalent pressure given by (54) and \( \kappa_i \) is defined as follows:

\[
\kappa_i = \frac{p_i}{p_{eq.max}} = \frac{p[\alpha_i, \beta_i]}{p_{eq.max}}
\]

7. ANALYSIS OF SOME TYPICAL RO-RO DECK STRUCTURES

In the following it has been investigated the influence of the longitudinal distribution of wheeled loads on girder and transverse stresses, in order to highlight the “plate effect” which re-distributes the load peaks on transverses, unlike the isolated beam scheme.

Two decks are analyzed: the first one is relative to a fast ferry, the second one to a Ro-ro Panamax ship (see Campanile et al., 2007).

7.1 ANALYSIS OF A RO-RO FAST FERRY DECK

It has been carried out the evaluation of the stresses acting on the primary supporting members of a fast ferry used to carry vehicles: the ship main dimensions are: \( L_{bp} = 97.61\ m; B = 17.10\ m; D = 10.40\ m; \Delta = 1420\ t.\) All transverses and girders have a \( 320x10+150x15 \) T section, while longitudinals are 60x6 offset bulb plates, in high-strength steel with \( \sigma_{yield}=355\ N/mm^2.\)

The data assumed in the analysis are:

- \( L_X=80\ m; \)
- \( l=L_Y=16\ m; \)
- \( s_X = 2\ m; \)
- \( s_Y = 2\ m; \)
- \( s_{XX} = 0.5\ m; \)
- \( t = 8\ mm; \)
- \( X_1 = 3000\ mm; \)
- \( X_2 = 2200\ mm; \)
- \( Q_X = 1.2\ t; \)
- \( n_Y = 7; \)
- \( \alpha_Y = 0.909\ g; \)
- \( I_{XX} = 29146\ cm^4; \)
- \( I_{YY} = 29067\ cm^4; \)
- \( I_{XY} = 5190\ cm^4; \)
- \( I_{YX} = 5359\ cm^4; \)
- \( r_{Xf} = 28.47\ cm; \)
- \( r_{Yf} = 28.38\ cm; \)
- \( r = 5; \)
- \( \eta_Y = 0.18. \)

In fig. 11 the deck scheme is shown.

From (53) the maximum static equivalent pressure is \( p_{eq.stat} = 2575\ N/m^2 \) so that, considering the vertical acceleration, the maximum total pressure is \( p_{eq.max} = 4914\ N/m^2. \) The longitudinal distribution of the equivalent pressure \( p \), and \( \sigma_{YSUP} \) stresses are listed in tab. 4 where:

- Transv. indicates the current transverse;
- \( X' \) is the distance in mm of the first axle respect to the current transverse in the interval \([\alpha_i, \beta_i];\)
- \( X'' \) is the distance in mm of the second axle (if present) respect to the current transverse in the interval \([\alpha_i, \beta_i];\)
- \( \kappa_i \) is the ratio between the pressure on the \( i\)-th transverse and the maximum one;
- \( \alpha_i \) indicates the aft limit, respect to the origin, of the \( i\)-th interval where \( p=p_i \) is continuous;
- \( \beta_i \) indicates the fore limit, respect to the origin, of the \( i\)-th interval;
- \( n_{ax} \) indicates the number of axles in the interval \([\alpha_i, \beta_i];\)
- \( k_{YF-Orth.} \) is the factor, determined by the orthotropic plate theory, to be inserted in (45.2) to determine the stress in the free flange of the \( i\)-th transverse with reference to \( p=p_{eq}; \)
- \( k_{YF-FEM} \) is the factor obtained by the FEM analysis of the corresponding structure.
The following diagrams show the equivalent pressure and the $\sigma_{Yf}$ stress longitudinal distribution.

\[ P_{eq} \text{ longitudinal distribution} \]

\[ \sigma_{Yf} \text{ longitudinal distribution} \]

where:

\[ k_1 = \frac{k_Y}{k_{Yf,max}} \quad (58) \]

This analysis shows that there is a significant redistribution of $\sigma_{Yf}$ stresses that can’t be evaluated by the isolated beam model. This effect unload the most loaded transverses and load the least loaded ones.

The maximum stresses on girders and transverses are:

- $\sigma_X = 100 \text{ N/mm}^2$
- $\sigma_Y = 163 \text{ N/mm}^2$

It is noticed that by a coarse mesh FEM analysis the following maximum stresses have been obtained:

- $\sigma_{X,FEM} = 115 \text{ N/mm}^2$
- $\sigma_{Y,FEM} = 175 \text{ N/mm}^2$

If the deck were loaded by the uniform pressure $p = 4914 \text{ N/m}^2$, equal to the maximum equivalent pressure, from fig. 4 it is obtained that the maximum stresses on girders and transverses would be:

- $k_{X,unif} = 0.0571 \rightarrow \sigma_{X,unif} = 141 \text{ N/mm}^2$
- $k_{Y,unif} = 0.0833 \rightarrow \sigma_{Y,unif} = 205 \text{ N/mm}^2$

Correspondingly by a coarse mesh FEM analysis, the following stresses have been obtained:
\[
\sigma_{Xf\text{-unif\,FEM}} = 146 \text{ N/mm}^2 \\
\sigma_{Yf\text{-unif\,FEM}} = 214 \text{ N/mm}^2
\]

Now, let us define the mean load parameter \( \chi \):

\[
\chi = \frac{\sum_{i=1}^{n} \chi_i}{n_f}
\]

(59)

which in the case under examination is 0.75.

As it occurs that:

\[
\psi \chi = \frac{\sigma_{Xf}}{\sigma_{Xf\text{-unif\,FEM}}} = 0.71
\]

(60.1)\[
\psi \chi = \frac{\sigma_{Yf}}{\sigma_{Yf\text{-unif\,FEM}}} = 0.80
\]

(60.2)

approximately the following positions can be done:

\[
k_{Xf} \equiv \chi \kappa_{Xf\text{-unif}}
\]

\[
k_{Yf} \equiv \chi \kappa_{Yf\text{-unif}}
\]

(61.1)\[
(61.2)
\]

Furthermore, in order to appreciate the roles of girders and transverses, the total external force work has been decomposed in three components, two of which have been associated to transverses and girders on the basis of the strain energy expression.

If the deck is loaded with a uniform equivalent pressure \( p \), the total external work is:

\[
L_p = \frac{1}{2} \int_{A} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 dA = \frac{8 \pi^4 p^2 b^4 a}{E_i} k_{girder}
\]

where:

\[
k_{girder} = \sum_{i=1}^{N} \sum_{m=1}^{M} \delta_{x_i - \delta_{x_{n,i}}} m^2 \int_0^1 \cos 2 \pi m \zeta \cos 2 \pi m \xi d \zeta
\]

(65)

The third term, similarly, is:

\[
L_{\text{transv}} = \frac{1}{2} \int_{A} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 dA = \frac{8 \pi^4 p^2 b^4 a}{E_i} k_{\text{transv}}
\]

where:

\[
k_{\text{transv}} = \sum_{i=1}^{N} \sum_{m=1}^{M} \delta_{y_i - \delta_{y_{n,i}}} m^2 \int_0^1 \cos 2 \pi m \zeta \cos 2 \pi m \xi d \zeta
\]

(66)

The second term is developed as follows:

\[
L_{\text{distors}} = \frac{1}{2} \int_{A} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 dA = 16 \pi^4 \frac{\eta_{\text{distors}} p^2 b^4 a}{E_i} k_{\text{distors}}
\]

where:

\[
k_{\text{distors}} = \sum_{i=1}^{N} \sum_{m=1}^{M} \delta_{x_i - \delta_{x_{n,i}}} m^2 \int_0^1 \cos 2 \pi m \zeta \cos 2 \pi m \xi d \zeta
\]

(67)

Applying these equations to the examined structure, it is obtained:

\[
L_{r} = 24610 \text{Nm}
\]

\[
L_{\text{girder}} = 1131 \text{Nm}
\]

\[
L_{\text{transv}} = 22857 \text{Nm}
\]

\[
L_{\text{distors}} = 622 \text{Nm}
\]

Corresponding percent ratios are:

\[
L_{girder} = 4.6\% \quad L_{\text{transv}} = 92.9\% \quad L_{\text{distors}} = 2.5\%
\]

It is apparent that transverses absorb the most part of the total external work; also the mean strain energy per unit length absorbed by each transversal supporting member is much greater than that one absorbed by girders:

\[
L_{\text{girder}} = \frac{1131}{7.80} = \frac{2N\text{m}}{m}
\]

\[
L_{\text{transv}} = \frac{22857}{39.16} = \frac{37N\text{m}}{m}
\]
7.2 ANALYSIS OF A RO-RO PANAMAX DECK

It has been carried out the evaluation of the highest stresses acting on the primary supporting members of a Ro-ro PANAMAX ship used to carry heavy vehicles; the ship main dimensions are: \( L_{bp} = 195.00 \text{ m}; \) \( B = 32.25 \text{ m}; \) \( D = 25.92 \text{ m}; \) \( \Delta = 44200 \text{ t}. \) Transverses and girders, have, respectively, \( 970 \times 11 + 320 \times 30 \) and \( 970 \times 12 + 280 \times 30 \) T sections, while longitudinals are \( 240 \times 10 \) offset bulb plates, in high-strength steel with \( \sigma_{\text{yield}} = 355 \text{ N/mm}^2 \). The data assumed in the analysis are:

- \( L_X = 160 \text{ m}; \)
- \( l = L_Y = 24 \text{ m}; \)
- \( s_X = 4 \text{ m}; \)
- \( s_Y = 2.463 \text{ m}; \)
- \( s_{ex} = 0.667 \text{ m}; \)
- \( t = 14 \text{ mm}; \)
- \( a_Z = 0.411 \text{ g}; \)
- \( n_V = 8; \)
- \( I_{ex} = 967698 \text{ cm}^4; \)
- \( I_{ey} = 911559 \text{ cm}^4; \)
- \( I_{px} = 244515 \text{ cm}^4; \)
- \( r_{xf} = 83.66 \text{ cm}; \)
- \( r_{yf} = 75.30 \text{ cm}; \)
- \( \rho = 7.41; \)
- \( \eta_t = 0.22. \)

The deck scheme is shown in fig. 14.

The reference vehicle has the main dimensions and the static axles loads shown in fig. 15.

The maximum total pressure is \( p_{eq,\text{max}} = 48647 \text{ N/m}^2 \). The longitudinal distribution of the equivalent pressure is shown in tab. 5.

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The maximum stresses on girders and transverses are:
- $\sigma_{Xf} = 154 \text{ N/mm}^2$
- $\sigma_{Yf} = 176 \text{ N/mm}^2$

If the deck were loaded by the uniform pressure $p = p_{eq,max} = 48647 \text{ N/m}^2$, the maximum stresses on girders and transverses would be:
- $k_{Xf-SUP-unif} = 0.0571 \rightarrow \sigma_{Xf-unif} = 446 \text{ N/mm}^2$
- $k_{Yf-SUP-unif} = 0.0833 \rightarrow \sigma_{Yf-unif} = 475 \text{ N/mm}^2$

so obtaining:

$$\psi_X = \frac{\sigma_{Xf}}{k_{Xf-SUP-unif}} = \frac{k_{Xf}}{k_{Xf-SUP-unif}} = 0.35 \quad (68.1)$$

$$\psi_Y = \frac{\sigma_{Yf}}{k_{Yf-SUP-unif}} = \frac{k_{Yf}}{k_{Yf-SUP-unif}} = 0.37 \quad (68.2)$$

As in this case $\chi = 0.35$ -see equation (59)-, approximately the positions (61.1) and (61.2) can be done, too. Concerning the strain energy components it is obtained:

$$L_t = 306225 N\text{m}$$
$$L_{girder} = 18624 \text{Nm}$$
$$L_{transv} = 280462 \text{Nm}$$
$$L_{distors} = 7139 \text{Nm}$$

Corresponding percent values are:

- $L_{girder} = 6.0\%$ - $L_{transv} = 91.6\%$ - $L_{distors} = 2.4\%$

The mean strain energies per unit length absorbed by each transverse and each girder are:

$$l_{girder} = \frac{18624}{5 \cdot 160} = 0.22 \text{ Nm}$$
$$l_{transv} = \frac{280462}{64 \cdot 24} = 183 \text{ Nm}$$

---

The following diagrams show the equivalent pressure and $k_{Yf}$ longitudinal distribution.

---

| 39 | 22.58 | 22.58 | 0 | 1335 | 25 | 0 | 0.89 |
| 40 | 22.58 | 0 | 0 | 2438 | 0 | 0 | 0.01 |
| 41 | 22.58 | 0 | 384 | 0 | 0 | 0.52 |
| 42 | 22.58 | 11.29 | 0 | 2078 | 1566 | 0 | 0.21 |
| 43 | 11.29 | 0 | 897 | 0 | 0 | 0.20 |
| 44 | 0 | 0 | 0 | 0 | 0 | 0.00 |
| 45 | 11.29 | 22.58 | 0 | 20 | 1620 | 0 | 0.52 |
| 46 | 11.29 | 22.58 | 22.58 | 2443 | 843 | 517 | 0.89 |
| 47 | 22.58 | 0 | 0 | 1946 | 0 | 0 | 0.13 |
| 48 | 22.58 | 0 | 876 | 0 | 0 | 0.40 |
| 49 | 22.58 | 11.29 | 0 | 1587 | 2058 | 0 | 0.27 |
| 50 | 11.29 | 0 | 0 | 405 | 0 | 0 | 0.26 |
| 51 | 0 | 0 | 0 | 0 | 0 | 0.00 |
| 52 | 11.29 | 22.58 | 0 | 511 | 2111 | 0 | 0.33 |
| 53 | 11.29 | 22.58 | 22.58 | 1952 | 352 | 1008 | 0.96 |
| 54 | 22.58 | 0 | 0 | 1455 | 0 | 0 | 0.25 |
| 55 | 22.58 | 0 | 1367 | 0 | 0 | 0.27 |
| 56 | 22.58 | 0 | 0 | 1096 | 0 | 0 | 0.34 |
| 57 | 11.29 | 0 | 86 | 0 | 0 | 0.30 |
| 58 | 11.29 | 0 | 2377 | 0 | 0 | 0.01 |
| 59 | 11.29 | 0 | 1003 | 0 | 0 | 0.18 |
| 60 | 11.29 | 22.58 | 22.58 | 1460 | 140 | 1500 | 0.95 |
| 61 | 22.58 | 22.58 | 0 | 2323 | 963 | 0 | 0.41 |
| 62 | 22.58 | 0 | 1859 | 0 | 0 | 0.15 |
| 63 | 22.58 | 0 | 604 | 0 | 0 | 0.46 |
| 64 | 11.29 | 0 | 578 | 0 | 0 | 0.24 |
8. PRELIMINARY DIMENSIONING OF RO-RO DECK PRIMARY SUPPORTING MEMBERS

Previous analyses have shown that the effective wheeled load distribution, expressed by means of the mean load parameter $\chi$, has great influence on the loading of girders and transverses. Particularly, it has been observed that transverses absorb the great part of the load, while girders contribute to a re-distribution of stresses, unloading the most loaded transverses and loading the least loaded ones.

In a previous work, see [6], a procedure for dimensioning of girders and transverses on the basis of the orthotropic plate theory has been proposed, considering a uniform pressure on deck and so neglecting the effective load longitudinal distribution.

From the numerical results of sections 7.1 and 7.2 it seems appropriate to assume for the pressure the mean equivalent pressure load $\chi p_{eq\text{max}}$. Moreover, as for garage decks the aspect ratio $r$ is much greater than 1, it is possible to assume $k_{Yf-SUP} = 0.0833$ and $k_{Xf-SUP} = 0.0571$.

Indicating with $\sigma_{all\_tr.}$ and $\sigma_{all\_long.}$ the allowable stresses for transverses and girders respectively, and with $p_{eq\text{max}}$ the maximum pressure transmitted by wheels according to equation (54) it’s possible to calculate the section modulus for transverses by the following relation:

$$W_{\mathsf{YMN}} \geq \chi \frac{0.0833 \cdot p_{eq\text{max}} \cdot L_Y^2 \cdot s_Y}{\sigma_{all\_tr.}}$$

where $p_{eq\text{max}}$ is in N/m$^2$, $L_Y$ and $s_Y$ in m, $\sigma_{all\_tr.}$ in N/mm$^2$ and $W_{\mathsf{YMN}}$ in cm$^3$. The modulus is inclusive of plating effective breadth $b_{Yf}$.

The condition valid for girders is:

$$W_{\mathsf{eXf}} \geq \chi \frac{0.0033 \cdot p_{eq\text{max}}^2 \cdot L_Y^3 \cdot s_X \cdot s_Y}{I_{Yf} \cdot \sigma_{all\_long.}^2}$$

where $p_{eq\text{max}}$ is in N/m$^2$, $L_Y$, $s_Y$ and $s_X$ in m, $I_{Yf}$ in cm$^4$, $r_{Xf}$ in cm, $\sigma_{all\_long.}$ in N/mm$^2$ and $W_{\mathsf{eXf}}$ in cm$^3$.

9. CONCLUSIONS

In this work the orthotropic rectangular plate bending equation with all edges clamped has been solved adopting the Rayleigh-Ritz method. Numerical calculations have been systematically performed in case of uniform pressure, varying two non-dimensional parameters, namely the virtual side ratio and the torsional coefficient. Response non-dimensional parameters, in terms of maximum deflection and maximum stresses, are given in a series of charts for their easy application. Some comparisons with well known published data and FEM analyses give a validation to the method.

The method has been applied to ro-ro garage decks, taking into account in this case a load variable along the deck length, according to the geometrical and mass characteristics of the reference vehicle. Two typical ro-ro ships have been examined. It has been highlighted that transverse beams absorb the most part of the external work done by the pressure load, as it could be expected. Besides, it has been found that there is an appreciable re-distribution of the load, so that almost the same maximum stresses are obtained considering simply the mean pressure acting uniformly on the deck; then those stresses can be evaluated directly by the orthotropic plate charts.

From that, the suggestion for a simple procedure for the preliminary dimensioning of ro-ro deck primary supporting members is given.

10. REFERENCES
